THE COLLEGES OF OXFORD UNIVERSITY

MATHEMATICS

Specimen of Written Test at Interview

Issued May 2002

This paper contains questions taken from recent tests and shows the general format. It should not be assumed that the range of topics covered will be the same in December 2001 but the format (and mark allocations) will be similar.

Time Allowed: 2½ hours

For candidates applying for Mathematics, Mathematics & Statistics, Computer Science, Mathematics & Computer Science, Mathematics & Philosophy

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Write your name, college (where you are sitting the test), and proposed course (from the list above) in BLOCK CAPITALS.

NAME:

COLLEGE:

COURSE:

Attempt all the questions. Each of the ten parts of Question 1 is worth 4 marks, and Questions 2,3,4,5 are worth 15 marks each, giving a total of 100.

Question 1 is a multiple choice question for which marks are given solely for the correct answers. Answer Question 1 on the grid on Page 2. Write your answers to Questions 2,3,4,5 in the space provided, continuing on the back of this booklet if necessary.

THE USE OF CALCULATORS OR FORMULA SHEETS IS PROHIBITED.

1. For each part of the question on pages 3–7 you will be given four possible answers, just one of which is correct. Indicate for each part **A–J** which answer (a), (b), (c), or (d) you think is correct with a tick (✓) in the corresponding column in the table below. You may use the spaces between the parts for rough working.

	(a)	(b)	(c)	(d)
A				
B				
C				
D				
E				
F				
G				
H				
I				
J				

A. The number of solutions of the equation

$$x^3 + ax^2 - x - 2 = 0$$

for which $x > 0$ is

(a) 1 (b) 2 (c) 3 (d) dependent on the value of a.

B. Of the following three alleged algebraic identities, at least one is wrong.

(i) $yz(z-y) + zx(x-z) + xy(y-x) = (z-y)(x-z)(y-x)$

(ii) $yz(z-y) + zx(x-z) + xy(y-x) = (z-y)(z-x)(y-x)$

(iii) $yz(z+y) + zx(z+x) + xy(y+x) = (z+y)(z+x)(y+x)$.

Which of the following statements is correct?

(a) Only identity (i) is right

(b) Only identity (ii) is right

(c) Identities (ii) and (iii) are right

(d) All these identities are wrong.

C. A child is presented with the following lettered tiles: M A M M A L. The number of different "words" he can make using all six tiles is

(a) 6 (b) 30 (c) 60 (d) 120.

D. Let $f(x)$ be the function $e^{e^{e^x}}$. The value of $f'(x)$ when $x = \ln 3$ is which of the following?

(a) $3e^{e^3}$ (b) $3e^{e^3+3}$ (c) e^{3e+e^3} (d) $9e^{e^3+1}$.

E. Which of the following integrals has the greatest value?

(a) $\int_0^{\pi/2} \sin^2 x \cos x \, dx$

(b) $\int_0^{\pi} \sin^2 x \cos x \, dx$

(c) $\int_0^{\pi/2} \sin x \cos^2 x \, dx$

(d) $\int_0^{\pi/2} \sin 2x \cos x \, dx$.

F. Observe that $2^3 = 8$, $2^5 = 32$, $3^2 = 9$ and $3^3 = 27$. From these facts, we can deduce that $\log_2 3$, the logarithm of 3 to base 2, is

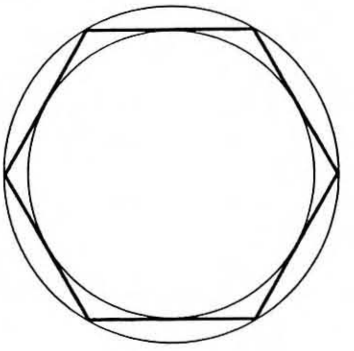
(a) between $1\frac{1}{3}$ and $1\frac{1}{2}$

(b) between $1\frac{1}{2}$ and $1\frac{2}{3}$

(c) between $1\frac{2}{3}$ and 2

(d) none of the above.

G. The figure shows a regular hexagon with its circumscribed and inscribed circles. What is the ratio of the area of the two circles?



(a) 4 : 3 (b) 6 : 5 (c) 7 : 5 (d) $\sqrt{3} : 2$

H. Aris, Boris, Clarice and Doris have to decide who will do the washing up. They decide to throw a fair 6-sided die: if it lands showing a 5 or 6, Aris will wash up; otherwise they throw again. The second time, if the result is a 5 or 6, Boris will wash up; otherwise they throw one last time. The final time, if the result is a 5 or 6, Clarice washes up, and otherwise it's Doris. (Of course, this is not a fair procedure!) Of the four, who is *second* most likely to do the washing up?

(a) Aris (b) Boris (c) Clarice (d) Doris.

I. The fixed positive integers a, b, c, d are such that exactly two of the following four statements are valid:

(i) $a \leqslant b < c \leqslant d$

(ii) $a + b = c + d$

(iii) $a = c$ and $b = d$

(iv) $ad = bc$.

You are also told that (ii) and (iv) is not the pair of valid statements. Which of the following must be the pair of valid statements?

(a) (i) and (ii)

(b) (i) and (iii)

(c) (i) and (iv)

(d) (iii) and (iv).

J. Just one of the following expressions is equal to $\sin 5\alpha$ for all values of α. Which one is it?

(a) $5\sin\alpha - 20\sin^3\alpha + 16\sin^5\alpha$

(b) $5\sin\alpha - 20\sin^3\alpha + 14\sin^5\alpha$

(c) $5\sin\alpha - 10\sin^2\alpha + 10\sin^3\alpha - 5\sin^4\alpha + \sin^5\alpha$

(d) $\sin\alpha - 5\sin^2\alpha + 10\sin^3\alpha - 10\sin^4\alpha + 5\sin^5\alpha$.

2. Suppose that the equation

$$x^4 + Ax^2 + B = (x^2 + ax + b)(x^2 - ax + b)$$

holds for all values of x.

(i) Find A and B in terms of a and b.

(ii) Use this information to find a factorization of the expression

$$x^4 - 20x^2 + 16$$

as a product of two quadratics in x.

(iii) Show that the four solutions of the equation

$$x^4 - 20x^2 + 16 = 0$$

can be written as $\pm\sqrt{7} \pm \sqrt{3}$.

3. Let

$$f(x) = \left(c - \frac{1}{c} - x\right)(4 - 3x^2),$$

where c is a positive constant and x varies over the real numbers.

(i) Show that $f(x)$ has one maximum and one minimum.

(ii) Show that the difference between the values of $f(x)$ at its turning points is

$$\frac{4}{9}\left(c + \frac{1}{c}\right)^3.$$

(iii) What is the least value that the difference in (ii) can have for $c > 0$?

Turn Over

4. The *absolute value* $|x|$ of a real number x is defined by

$$|x| = \begin{cases} x & \text{if } x \geqslant 0, \\ -x & \text{if } x < 0. \end{cases}$$

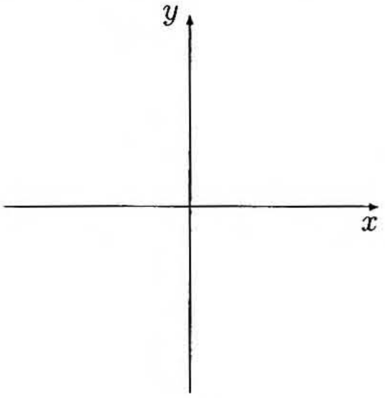
(So, for example, $|3| = 3$ and $|-5| = 5$.)

(i) Sketch the locus of points (x, y) in the first quadrant ($x \geqslant 0$, $y \geqslant 0$) of the x-y plane that satisfy the equation $x + y = 1$.

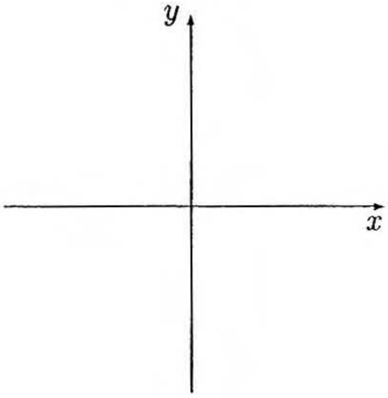
(ii) Hence, sketch the set of points (x, y) in the whole x-y plane that satisfy the equation $|x| + |y| = 1$.

(iii) Give the equations of the straight lines that contain the sides of the figure that you have drawn in (ii).

(iv) If $|x| + |y| = 1$, what are the maximum and minimum values of $\sqrt{x^2 + y^2}$? At which points (x, y) are they attained?

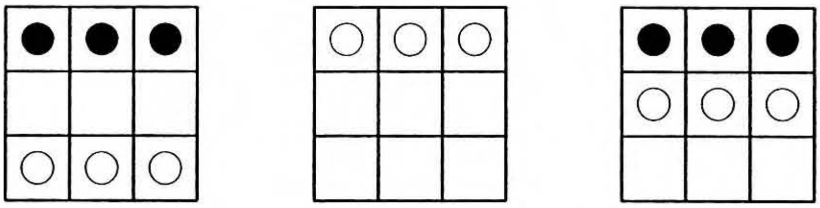


(i)



(ii)

5. The game of *Hexaglide* is played on a board with 3×3 squares; White and Black each have 3 pieces, and they begin as shown in the first diagram. White moves first, and the players take turns to move one of their pieces forwards or backwards to an adjacent empty square. (Pieces never move sideways or diagonally, and they are never captured or removed from the board.)



(i) In a practice game, White plays without any Black pieces on the board, and, from his usual starting position, reaches the position shown in the second diagram. How many different sequences of moves end with this position if White makes (a) 6 moves, (b) 7 moves, (c) 8 moves?

For parts (ii) *and* (iii) *of this question, give 'yes' or 'no' answers in the grids opposite; for both parts, you need not show your working or explain your answers.*

In the real game, White and Black both play, and a player wins if he can trap his opponent's pieces so that they cannot move: the third diagram shows a win for White.

(ii) (a) Is it possible to reach the position shown in the third diagram?
(b) Is it possible to reach a position where Black has won?
(c) Can White play so as to ensure that either he wins or the game goes on forever?
(d) Can Black play so as to ensure that either he wins or the game goes on forever?

(iii) In an advanced version of the game, the board has 4×4 squares, and each player has 4 pieces. What would be the answers to the four questions in part (ii) in this case?

(a)	
(b)	
(c)	
(d)	

(ii)

(a)	
(b)	
(c)	
(d)	

(iii)